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The kinetic equation for plasma with arbitrary strong interaction in a weak electric field

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Abstract. The exact linearized kinetic equation for plasma in a weak electric field of arbitrary frequency is obtained. All effects of interaction are included in the collision integral with the help of an equilibrium correlation function. The perturbation theory is considered for this function and a generalized solution for low-frequency and high-frequency collisions is obtained.

1. Introduction

At present there is a great variety of experimental information on the static and high-frequency conductivity of gaseous and metallic plasmas and of some other plasma systems (Noskov 1983, Ebeling 1983, Fortov and Iakubov 1984, Kovalenko *et al* 1990). However, the available theoretical results cover, mainly, two frequency ranges: $\omega \to 0$ and $\omega \gg \bar{\nu}$, where ω is the external electric field frequency and $\bar{\nu}$ is the average collision frequency. Thus, the necessity arises to describe the conductivity in the frequency range $\omega \sim \bar{\nu}$. In this case one should take into account the frequency dependence of conductivity connected with the parameters $\hbar\omega/k_B T$, $\hbar\omega/\bar{\varepsilon}$, ω/ω_p, etc, where T is the temperature of the system, $\bar{\varepsilon}$ is the characteristic energy of particles and ω_p is the plasma frequency. The problem of describing the conductivity of the charged-particle systems (Coulomb systems, CSs) at arbitrary frequencies ω of the external field becomes especially important in connection with intense study of the laser radiation effects on metals, non-ideal plasma and other plasma systems.

In the theoretical study of conductivity, two methods are mainly used: the kinetic equation (KE) method (Kadanoff and Baym 1962, Silin 1971, Klimontovich 1975) and the theory of linear response (TLR) (Zubarev 1971, Akhiezer and Peletminsky 1977). The difference between the above two methods is caused, in particular, by the different order of applying perturbation theory with respect to interparticle interaction and with respect to the applied electric field. In the KE method, first the splitting of a non-equilibrium high-order correlation function to lower order is done, using perturbation theory with respect to interparticle interaction (or to the density of scatterers), and then the obtained closed KE for the single-particle distribution function is linearized with respect to the electric field. In the TLR, firstly, linearization with respect to the external field is carried out, making it possible to express the conductivity through equilibrium correlation functions which strictly take into account both the interparticle interaction and the spatial and frequency dispersion. Then, based on diagram methods of perturbation theory, approximate expressions are derived. It is clear that in a consistent theory the order of the procedures used should not affect the final results.

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To describe the CS in a weak electric field it is natural to prefer the TLR since, being based on diagram methods of calculation of temperature Green functions (Abrikosov *et al* 1962), the important possibilities arise of including the effects of CS non-ideality and comparatively strict evaluation of the approximations used. In parallel with this, it is possible to verify by means of the TLR the validity of expressions derived in the KE method, which is of great importance. This is connected with the fact that traditional schemes of deducing the KE are based on the hypothesis that high-order non-equilibrium correlation functions in the case of a weak interaction can be uncoupled into low-order ones, a realization that cannot be argued exactly (Silin 1971, Klimontovich 1975). An intuitive element is conserved when the KE is deduced in a more consistent method due to Kadanoff and Baym (1962).

However, certain hopes connected with the TLR have not yet been completely realized due to considerable difficulties in solution of the diagram equations obtained, because of their complexity. It could even be said that at present only the simplest model of electrons in a field of immovable weak scatterers has been considered in the TLR while a consistent approach to calculation of the weakly non-ideal plasma conductivity is absent.

It should be noted that during recent years some attempts were made to write the general linearized KE for the CS on the basis of the memory function theory (Forster 1975). Using this approach it may be possible to compose a consistent perturbation theory for the consideration of CS kinetics in an external field. However, the exact expression for the generalized collision integral in the linearized KE has not yet been derived.

Here, the exact linearized KE for the CS in an external electric field, holding true for arbitrary strong interaction between particles and for arbitrary frequencies of the external field, is deduced. Some consequences of applying the perturbation theory for the solution of this KE are considered and briefly discussed. There is an agreement with the KE method results for a weakly non-ideal plasma in the range of low and high frequencies of the field.

2. Single-particle distribution function for a CS in a weak electric field

Let us consider an electrically neutral CS,

$$\sum_a z_a e n_a = 0 \tag{1}$$

in a weak electric field set by the scalar potential $U^{\text{ext}}(\boldsymbol{r}, t)$. In equation (1) n_a is the average density of particles of type a with charge $z_a e$ and mass m_a. The Wigner distribution function $f_a(\boldsymbol{r}, \boldsymbol{R}, t)$ of these particles can be represented by

$$\begin{gathered} f_a(\boldsymbol{r}, \boldsymbol{R}, t) = \mathrm{Sp}(\hat{\hat{F}}(t)\hat{\Psi}_a^+(\boldsymbol{r}_1, t)\hat{\Psi}_a(\boldsymbol{r}_2, t)) \\ \boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1 \qquad \boldsymbol{R} = \tfrac{1}{2}(\boldsymbol{r}_2 + \boldsymbol{r}_1) \end{gathered} \tag{2}$$

where $\hat{\Psi}_a^+(\boldsymbol{r}, t)$ and $\hat{\Psi}_a(\boldsymbol{r}, t)$ are the creation and annihilation field operators of the particles in the Heizenberg representation,

$$\hat{\hat{F}}(t) = \exp(\mathrm{i}\hat{H}t/\hbar)\hat{F}(t)\exp(-\mathrm{i}\hat{H}t/\hbar). \tag{3}$$

$\hat{F}(t)$ is the non-equilibrium statistical CS operator in an external electric field satisfying the motion equation

$$i\hbar\frac{\partial \hat{F}(t)}{\partial t}=[\hat{H}+\hat{V}^{\text{ext}}(t),\hat{F}(t)] \tag{4}$$

with the initial condition

$$\hat{F}(-\infty)=\hat{F}_0.$$

$\hat{F}_0$ is the statistical Gibbs operator formed on the exact Hamiltonian $\hat{H}$ of the CS in the absence of an external field and characterized by temperature T, chemical potentials μ_a and volume V,

$$\hat{H}=\sum_a\sum_p \varepsilon_p^a \hat{N}_p^a+\frac{1}{2V}\sum_q u_{ab}(q)(\hat{n}_q^a\hat{n}_{-q}^b-\hat{N}_a\delta_{a,b}) \tag{5}$$

$V^{\text{ext}}(t)$ is the Hamiltonian of interaction between the CS and an external electric field,

$$\hat{V}^{\text{ext}}(t)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3q}{(2\pi)^3}\exp(-i\omega t)\hat{\rho}_{-q}U^{\text{ext}}(\boldsymbol{q},\omega) \tag{6}$$

$U^{\text{ext}}(q,\omega)$ is the Fourier component of the scalar potential,

$$\begin{aligned}&\hat{N}_p^a=\hat{a}_p^+\hat{a}_p \qquad \hat{n}_{pq}^a=\hat{a}_{p-q/2}^+\hat{a}_{p+q/2}\\ &\hat{n}_q^a=\sum_p\hat{n}_{pq}^a \qquad \hat{\rho}_q=\sum_a z_a e\hat{n}_q^a \qquad \hat{N}_a=\sum_p\hat{N}_p^a\end{aligned} \tag{7}$$

$\hat{a}_p^+$ and $\hat{a}_p$ are the creation and annihilation operators of particles of type a with momentum $\hbar\boldsymbol{p}$ (here and below the spin indices are omitted), $u_{ab}(q)$ is the Fourier component of the Coulomb interaction potential

$$u_{ab}(q)=\frac{4\pi z_a z_b e^2}{q^2}. \tag{8}$$

Using the TLR (Zubarev 1971, Akhiezer and Peletminsky 1977), it is easy to derive the correction to the equilibrium single-particle distribution function $f_a^{(0)}(\boldsymbol{r})$ of the uniform and isotropic CS which is proportional to an external electric field (Bobrov *et al* 1989, Bobrov and Trigger 1990):

$$f_a^{(1)}(\boldsymbol{r},\boldsymbol{R},t)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3p}{(2\pi)^3}\exp(-i\omega t+i\boldsymbol{pr}+i\boldsymbol{kR})f_a^{(1)}(\boldsymbol{p},\boldsymbol{k},\omega) \tag{9}$$

$$f_a^{(1)}(\boldsymbol{p},\boldsymbol{k},\omega)=\phi_a^{\text{R}}(\boldsymbol{p},\boldsymbol{k},\omega)U^{\text{ext}}(\boldsymbol{k},\omega) \tag{10}$$

where the retarded equilibrium Green function $\phi_a^{\text{R}}(\boldsymbol{p},\boldsymbol{k},\omega)$ is

$$\phi_a^{\text{R}}(\boldsymbol{p},\boldsymbol{k},\omega)=-\frac{i}{\hbar}\int_0^{\infty}dt\,\exp(i\omega t)\langle[\hat{n}_{pk}^a(t),\hat{\rho}_{-k}(0)]\rangle\equiv\langle\langle\hat{n}_{pk}^a|\hat{\rho}_{-k}\rangle\rangle_\omega \tag{11}$$

$$\langle\ldots\rangle=\text{Sp}\,\hat{F}_0(\ldots). \tag{12}$$

It is evident that

$$\phi_a^{\text{R}}(\boldsymbol{p},\boldsymbol{k},\omega)=\sum_b z_b e\phi_{ab}^{\text{R}}(\boldsymbol{p},\boldsymbol{k},\omega) \tag{13}$$

$$\phi_{ab}(\boldsymbol{p},\boldsymbol{k},\omega)=\langle\langle\hat{n}_{pk}^a|\hat{n}_{-k}^b\rangle\rangle_\omega. \tag{14}$$

Thus, to determine the linearized distribution function $f_a^{(1)}(\boldsymbol{p}, \boldsymbol{k}, \omega)$ it is sufficient to calculate the retarded Green function $\phi_a^{\mathrm{R}}(\boldsymbol{p}, \boldsymbol{k}, \omega)$. However, the retarded Green functions are the analytical continuation of the respective temperature Green functions from discrete points to the upper half-plane of complex ω (Abrikosov *et al* 1962). To calculate the temperature Green functions there exist well-developed diagram methods of perturbation theory with respect to interparticle interaction (Abrikosov *et al* 1962, Kadanoff and Baym 1962, Kraeft *et al* 1986). In particular, for the temperature Green function (figure 1)

$$\phi_a^T(\boldsymbol{p}, \boldsymbol{k}, \mathrm{i}\Omega_n) = -\tfrac{1}{2}\int_{-1/k_BT}^{1/k_BT} \mathrm{d}\tau \exp(\mathrm{i}\Omega_n\tau)\langle \hat{T}_\tau(\hat{n}_{pk}^a(-\mathrm{i}\hbar\tau)\hat{\rho}_{-k}0)\rangle \equiv \langle\langle \hat{n}_{pk}^a | \hat{\rho}_{-k}\rangle\rangle_{\mathrm{i}\Omega_n} \qquad (15)$$

where $\hat{T}_\tau$ is the operator of τ-ordering and $\Omega_n = 2\pi k_B Tn$ ($n = 0, \pm 1 \ldots$) one can write the diagram expansion (figure 2) and identify the component (figure 3)

$$\bar{\phi}_a^T(\boldsymbol{p}, \boldsymbol{k}, \mathrm{i}\Omega_n) = \langle\langle \hat{n}_{pk}^a | \hat{\rho}_{-k}\rangle\rangle_{\mathrm{i}\Omega_n}^{(k)} \qquad (16)$$

which is irreducible in the 'k-channel' along a single line of the Coulomb interaction:

$$u(k) = \frac{4\pi}{k^2}.$$

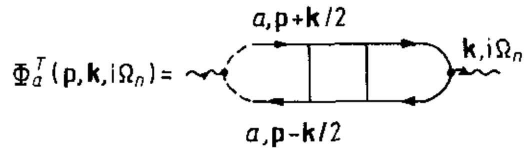

Figure 1.

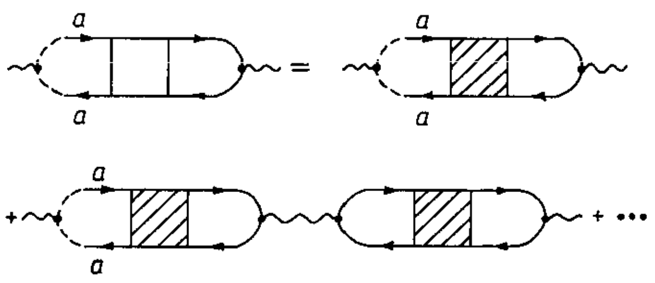

Figure 2.

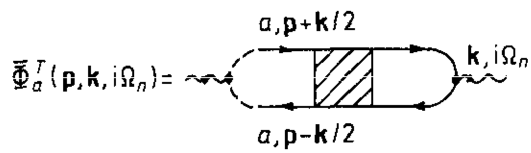

Figure 3.

The function $\Pi(k, \mathrm{i}\Omega_n)$ (figure 4) represents the polarization operator

$$\Pi(k, \mathrm{i}\Omega_n) = \sum_a z_a e \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \bar{\phi}_a^T(\boldsymbol{p}, \boldsymbol{k}, \mathrm{i}\Omega_n) = V^{-1} \langle\langle \hat{\rho}_k | \hat{\rho}_{-k} \rangle\rangle_{\mathrm{i}\Omega_n}^{(k)} \tag{17}$$

$$\Pi(k, \mathrm{i}\Omega_n) = \sum_{ab} z_a z_b e^2 \Pi_{ab}(k, \mathrm{i}\Omega_n) \tag{18}$$

$$\Pi_{ab}(k, \mathrm{i}\Omega_n) = V^{-1} \langle\langle \hat{n}_k^a | \hat{n}_{-k}^b \rangle\rangle_{\mathrm{i}\Omega_n}^{(k)}. \tag{19}$$

The polarization operator $\Pi(k, \mathrm{i}\Omega_n)$ is the irreducible component along a single interaction line $u(k)$ in the 'k-channel' of the Green function,

$$L^T(k, \mathrm{i}\Omega_n) = V^{-1} \langle\langle \hat{\rho}_{-k} | \hat{\rho}_{-k} \rangle\rangle_{\mathrm{i}\Omega_n}. \tag{20}$$

The definitions (11)–(20) should be understood in the thermodynamical limit: $V \to \infty$, $\langle \hat{N}_a \rangle \to \infty$, $n_a = \langle \hat{N}_a \rangle / V = \mathrm{const}$.

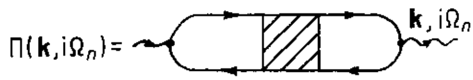

Figure 4.

The exact longitudinal dielectric function $\varepsilon^{\mathrm{l}}(k, \omega)$ of the CS is connected with the retarded Green function

$$L^{\mathrm{R}}(k, \omega) = V^{-1} \langle\langle \hat{\rho}_k | \hat{\rho}_{-k} \rangle\rangle_\omega \tag{21}$$

by the relation (Zubarev 1971, Akhiezer and Peletminsky 1977)

$$\varepsilon^{\mathrm{l}}(k, \omega) = (1 + u(k) L^{\mathrm{R}}(k, \omega))^{-1}. \tag{22}$$

Therefore

$$\varepsilon^{\mathrm{l}}(k, \mathrm{i}\Omega_n) = 1 - u(k)\Pi(k, \mathrm{i}\Omega_n). \tag{23}$$

Including equations (16)–(23), we obtain after analytical continuation (Bobrov *et al* 1989, Bobrov and Trigger 1990)

$$\phi_a^{\mathrm{R}}(\boldsymbol{p}, \boldsymbol{k}, \omega) = \frac{\bar{\phi}_a(\boldsymbol{p}, \boldsymbol{k}, \omega)}{\varepsilon^{\mathrm{l}}(k, \omega)} \tag{24}$$

or

$$f_a^{(1)}(\boldsymbol{p}, \boldsymbol{k}, \omega) = \bar{\phi}_a(\boldsymbol{p}, \boldsymbol{k}, \omega) U(\boldsymbol{k}, \omega) = \frac{\mathrm{i} k_\alpha \bar{\phi}_a(\boldsymbol{p}, \boldsymbol{k}, \omega)}{k^2} E_\alpha(\boldsymbol{k}, \omega). \tag{25}$$

Here $E_\alpha(\boldsymbol{k}, \omega) = -\mathrm{i} k_\alpha U(\boldsymbol{k}, \omega)$ is the electric field intensity in the medium and $U(\boldsymbol{k}, \omega)$ is the scalar potential,

$$U(\boldsymbol{k}, \omega) = \frac{U^{\mathrm{ext}}(\boldsymbol{k}, \omega)}{\varepsilon^{\mathrm{l}}(\boldsymbol{k}, \omega)}. \tag{26}$$

Thus, we have derived the exact relation (using diagram methods) between the CS linearized distribution function and the electric field intensity (potential) in the medium through the function $\bar{\phi}_a(\boldsymbol{p}, \boldsymbol{k}, \omega)$.

Induction in the CS electric current is determined by the expression

$$j_\alpha(\boldsymbol{k}, \omega) = \sum_a z_a e \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{\hbar p_\alpha}{m_a} f_a^{(1)}(\boldsymbol{p}, \boldsymbol{k}, \omega). \tag{27}$$

Therefore, for the longitudinal CS conductivity $\sigma^l(k, \omega)$ connecting the induced current with the electric field in the medium, we find

$$\sigma^l(k,\omega)=\frac{ik_\alpha}{k^2V}\langle\langle\hat{j}^\alpha_k|\hat{\rho}_{-k}\rangle\rangle^{(k)}_\omega \tag{28}$$

or, using the continuity equation in the operator form,

$$\sigma^l(k,\omega)=\frac{i\omega}{k^2}\Pi(k,\omega). \tag{29}$$

The Fourier component of the electric current $\hat{j}^\alpha_k$ of the CS in equation (28) is

$$\hat{j}^\alpha_k=\sum_a\sum_p\frac{\hbar z_a e p_\alpha}{m_a}\hat{n}^a_{pk}. \tag{30}$$

Comparing equations (23) and (29) results in the known expression (Silin and Rukhadze 1961) which relates the dielectric function with conductivity:

$$\varepsilon^l(k,\omega)=1+\frac{4\pi i}{\omega}\sigma^l(k,\omega). \tag{31}$$

Let us consider in more detail the form of conductivity and dielectric function of the CS in the long-wave limit $k\to 0$. There are actually two principle different long-wave limits: (i) $\omega/k\to 0$, $k\to 0$; (ii) $k/\omega\to 0$ (Silin and Rukhadze 1961).

For the below considerations, let us use the relation

$$\lim_{\omega\to 0}\langle\langle\hat{A}|\hat{B}\rangle\rangle_\omega=\langle\langle\hat{A}|\hat{B}\rangle\rangle_{i\Omega_n=0} \tag{32}$$

whose validity can easily be seen using the spectral representation for the retarded and temperature Green functions (Abrikosov *et al* 1962). An exception is the case when at least one of the operators $\hat{A}$ or $\hat{B}$ is the integral of motion.

(i) According to equations (1) and (32)

$$\lim_{\omega\to 0}\bar{\phi}_a(\boldsymbol{p},\boldsymbol{k},\omega)=\bar{\phi}^T_a(\boldsymbol{p},\boldsymbol{k},i\Omega_n=0). \tag{33}$$

Using the diagram expansions one can easily verify that the function $\bar{\phi}^T_a(\boldsymbol{p},\boldsymbol{k},i\Omega_n=0)$ has no singularities at small wavevectors $\boldsymbol{k}$.

Therefore

$$\lim_{k\to 0}\bar{\phi}^T_a(\boldsymbol{p},\boldsymbol{k},i\Omega_n=0)=\sum_b z_b e\langle\langle\hat{N}^a_p|\delta\hat{N}_b\rangle\rangle_{i\Omega_n=0} \tag{34}$$

where $\delta\hat{N}_a=\hat{N}_a-\langle\hat{N}_a\rangle$.

Hence,

$$\lim_{k\to 0}\bar{\bar{\phi}}_a(\boldsymbol{p},\boldsymbol{k},\omega/k\to 0)=\sum_b z_b e\langle\langle\hat{N}^a_p|\delta\hat{N}_b\rangle\rangle_{i\Omega_n=0}=-\sum_b z_b e\left(\frac{\partial f^{(0)}_a(p)}{\partial\mu_b}\right)_T. \tag{35}$$

Here $f^{(0)}_a(p)$ is the exact equilibrium single-particle distribution function of particles of type a. Thus, including equations (17), (23) and (29), we find (Klyuchnikov and Trigger 1990)

$$\lim_{\omega/k\to 0}\sigma^l(k,\omega)=0 \tag{36}$$

$$\langle R\rangle^{-2}=\lim_{k\to 0}\lim_{\omega/k\to 0}k^2(\varepsilon^l(k,\omega)-1)=\sum_{ab}4\pi z_a z_b e^2\left(\frac{\partial n_a}{\partial\mu_b}\right)_T. \tag{37}$$

Equation (36) means that the transition to the limit $\omega/k \to 0$ corresponds to considering the equilibrium in a static external field (absence of transport processes). The variable $\langle R \rangle$ in equation (37) characterizes the distance where a weakening of a test charge static field in a CS takes place.

(ii) To determine the function $\bar{\phi}_a(\boldsymbol{p}, \boldsymbol{k}, \omega)$ in the limit $k/\omega \to 0$ let us use the continuity equation in the operator form for the function $\bar{\phi}_a^T(\boldsymbol{p}, \boldsymbol{k}, \mathrm{i}\Omega_n)$ $(\Omega_n \neq 0)$, which results in

$$\bar{\phi}_a^T(\boldsymbol{p}, \boldsymbol{k}, \mathrm{i}\Omega_n) = \frac{z_a e}{\mathrm{i}\Omega_n}(f_a^{(0)}(\boldsymbol{p}-\boldsymbol{k}/2) - f_a^{(0)}(\boldsymbol{p}+\boldsymbol{k}/2)) + \frac{\hbar k_\alpha}{\mathrm{i}\Omega_n}\langle\langle \hat{n}^a_{\boldsymbol{pk}} | \hat{j}^\alpha_{-\boldsymbol{k}} \rangle\rangle^{(k)}_{\mathrm{i}\Omega_n} \tag{38}$$

(Bobrov *et al* 1989, Bobrov and Trigger 1990). The function $\langle\langle \hat{n}^a_{\boldsymbol{pk}} | \hat{j}^\alpha_{-\boldsymbol{k}} \rangle\rangle^{(k)}_{\mathrm{i}\Omega_n}$ at $\Omega_n \neq 0$ has no singularities in the range of small wavevectors $\boldsymbol{k}$. Hence, for isotropic and uniform CSs we have

$$\lim_{k\to 0} \langle\langle \hat{n}^a_{\boldsymbol{pk}} | \hat{j}^\alpha_{-\boldsymbol{k}} \rangle\rangle^{(k)}_{\mathrm{i}\Omega_n} = \langle\langle \hat{N}^a_{\boldsymbol{p}} | \hat{I}^\alpha \rangle\rangle_{\mathrm{i}\Omega_n} = \frac{p_\alpha}{p^2} \langle\langle \hat{N}^a_{\boldsymbol{p}} | p_\beta \hat{I}^\beta \rangle\rangle_{\mathrm{i}\Omega_n} \tag{39}$$

where

$$\hat{I}^\alpha = \hat{j}^\alpha_{\boldsymbol{k}=0} = \sum_a \sum_{\boldsymbol{p}} \frac{\hbar z_a e p_\alpha}{m_a} \hat{N}^a_{\boldsymbol{p}}$$

is the so-called total electric current operator.

Thus, after analytical continuation we obtain for the linearized distribution function of the CS at the limit $k/\omega \to 0$ (Bobrov *et al* 1989, Bobrov and Trigger 1990)

$$f_a^{(1)}(\boldsymbol{p}, \boldsymbol{k} \to 0, \omega) = \frac{\mathrm{i}}{\omega}\left(-\frac{z_a e}{\hbar p}\frac{\partial f_a^{(0)}}{\partial p}(p) + \frac{1}{p^2}\langle\langle \hat{N}^a_{\boldsymbol{p}} | p_\beta \hat{I}^\beta \rangle\rangle_\omega\right) p_\alpha E_\alpha(k \to 0, \omega). \tag{40}$$

It is easy to see that equations (27) and (40) directly result in the known Kubo formula (Kubo 1957) for the frequency dispersion of the conductivity:

$$\sigma(\omega) = \frac{\mathrm{i}\omega_p^2}{4\pi\omega} + \frac{\mathrm{i}}{3\omega V}\langle\langle \hat{I}^\alpha | \hat{I}^\alpha \rangle\rangle_\omega \tag{41}$$

where

$$\omega_p = \left(\sum_a \frac{4\pi z_a^2 e^2 n_a}{m_a}\right)^{1/2}$$

is the plasma frequency.

It is necessary to stress that the proposed derivation of (41) should be considered as a consistent derivation of the Kubo formula for the frequency dependence of the conductivity $\sigma(\omega) = \lim_{k\to 0} \sigma^l(k, \omega)$ which relates the induced current with a weakly non-uniform electric field in the medium instead of relating it with an external field as in the case of the traditional derivation (Kubo 1957).

Equations (40), (41) also hold true for systems including neutral particles interacting with each other and with charged particles through the short-range potentials, i.e. the case of the interaction potentials with their Fourier component remaining of finite value at small wavevectors.

The proposed derivation of (41) is based on the diagram method of the perturbation theory with respect to interparticle interaction. Thus, the validity of the Kubo formula (41) is, generally speaking, restricted by the range of divergence of the functional series in the perturbation theory. In this connection, let us mention an alternative approach to deriving the formula of the frequency dispersion of conductivity in the framework of the TLR without application of the diagram methods; this approach leads to principally different results (Bobrov and Trigger 1986, 1988).

In the case of the one-component CS (OCP), e.g. an electron fluid on a positive background, at $\omega \neq 0$

$$f_a^{(1)}(p, k\to 0, \omega) = -\frac{z_a e \mathrm{i}}{\hbar\omega}\frac{\partial f_a^{(0)}(p)}{\partial p_\alpha} E_\alpha(k\to 0, \omega) \tag{42}$$

$$\sigma^{\mathrm{OCP}}(\omega) = \frac{\mathrm{i}\omega_p^2}{4\pi\omega} \tag{43}$$

since for the OCP

$$\hat{I}^\alpha(t) = \hat{I}^\alpha(0). \tag{44}$$

3. Generalized kinetic equation for the linearized distribution function

We restrict our below consideration to examination of the action of a weakly non-uniform electric field on the CS, i.e. we shall assume that the electric field is slowly varying within characteristic scales: along the free path length or on the particle path length during the field variation period. This means considering the linearized distribution function in the limit $k/\omega \to 0$ (equation (40)). The calculation of this function is reduced to deriving the retarded Green function $\langle\langle \hat{N}_p^a | p_\alpha \hat{I}^\alpha \rangle\rangle_\omega$. For calculating the temperature Green function $\langle\langle \hat{N}_p^a | p_\alpha \hat{I}^\alpha \rangle\rangle_{\mathrm{i}\Omega_n}$ with application of perturbation theory to interparticle interaction, the series expansion is in the general case performed with respect to the following dimensionless parameters:

(i) The thermodynamical parameter of non-ideality

$$\gamma_{ab} \sim \frac{z_a z_b e^2 n^{1/3}}{\bar{\varepsilon}} \tag{45}$$

where $\bar{\varepsilon}$ is the average kinetic energy of particles.

(ii) The Born scattering parameter

$$\alpha_{ab} \sim \frac{z_a z_b e^2}{\hbar \bar{v}} \tag{46}$$

where $\bar{v}$ is the characteristic velocity of particles.

(iii) The dynamical parameter $\varkappa_{ab}$, which is characterized by the ratio of the characteristic collision frequency $\bar{\nu}$ to the external field frequency,

$$\varkappa_{ab} \sim \bar{\nu}_{ab}/\omega \qquad \bar{\nu}_{ab} \sim \gamma_{ab}^2 \bar{v} n^{1/3}. \tag{47}$$

Thus, to determine the linearized distribution function and the conductivity of the CS at low frequencies ω a necessity arises for summation of infinite series of perturbation theory with respect to the parameter $\varkappa_{ab}$, even in the case of a weakly non-ideal plasma†.

† Strictly speaking, there is one more dynamical parameter ω_p/ω which is connected with interparticle interaction. However, it is automatically taken into account when expansion with respect to the screened potentials of interaction is performed.

When calculating the Green function $\langle\langle \hat{N}_p^a | p_\alpha \hat{I}^\alpha \rangle\rangle_{i\Omega_n}$ there is an important possibility of effective summation with respect to the parameter $\varkappa_{ab}$ if diagram methods are used. Actually, the Green function $\langle\langle \hat{N}_p^a | p_\alpha \hat{I}^\alpha \rangle\rangle_{i\Omega_n}$ can be written in the form

$$\begin{aligned} \langle\langle \hat{N}_p^a | p_\alpha \hat{I}^\alpha \rangle\rangle_{i\Omega_n} &= \sum_b \sum_{p'} \frac{z_b e\hbar p_\alpha p'_\alpha}{m_b} C_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n) \\ C_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n) &= \langle\langle \hat{N}_p^a | \hat{N}_{p'}^b \rangle\rangle_{i\Omega_n}. \end{aligned} \tag{48}$$

The temperature Green function $C_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n)$ is the result of the summing up of the two-particle Green function $D_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n, i\omega_m)$ with respect to discrete frequencies $\omega_m = (2m+1)k_B T\pi$ (for fermions) or $\omega_m = 2mk_B T\pi$ (for bosons) (figure 5)

$$C_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n) = k_B T \sum_{\omega_m} D_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n, i\omega_m). \tag{49}$$

For the function $D_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n, i\omega_m)$ the diagram expansion (figure 6) can be written with extraction of the term $\Gamma_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n, i\omega_m, i\omega_l)$ (figure 7), which is irreducible with respect to two exact one-particle Green functions $g_a(\boldsymbol{p}, i\omega_m)$:

$$\begin{aligned} D_{ab}(\boldsymbol{p}, \boldsymbol{p}', i\Omega_n, i\omega_m) &= g_a(\boldsymbol{p}, i\omega_m + i\Omega_n) g_a(\boldsymbol{p}, i\omega_m)\delta_{a,b}\delta_{p,p'} + k_B T \sum_{\omega_l}\sum_c g_a(\boldsymbol{p}, i\omega_m + i\Omega_n) g_a(\boldsymbol{p}, i\omega_m) \\ &\quad \times \int \frac{d^3 p_1}{(2\pi)^3} \Gamma_{ac}(\boldsymbol{p}, \boldsymbol{p}_1, i\Omega_n, i\omega_m, i\omega_l) D_{cb}(\boldsymbol{p}_1, \boldsymbol{p}', i\Omega_n, i\omega_l). \end{aligned} \tag{50}$$

The diagram expansion for the function Γ_{ab} is presented in figure 8. It is easy to see that expansion of the function Γ_{ab} in a perturbation theory series is connected with the parameters α_{ab} and γ_{ab} only. Therefore, equation (50) makes it possible to sum up the perturbation theory series with respect to the parameter $\varkappa_{ab}$.

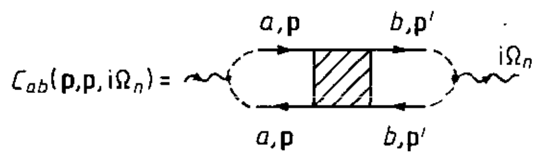

Figure 5.

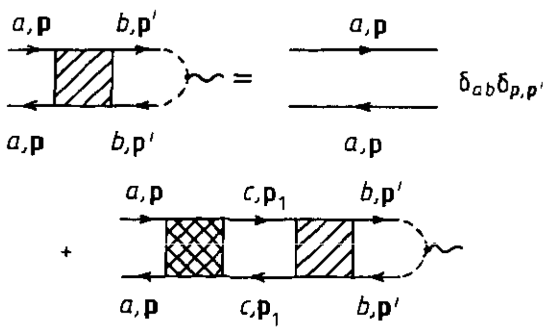

Figure 6.

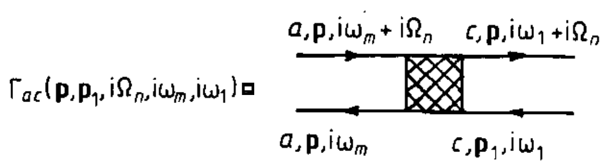

Figure 7.

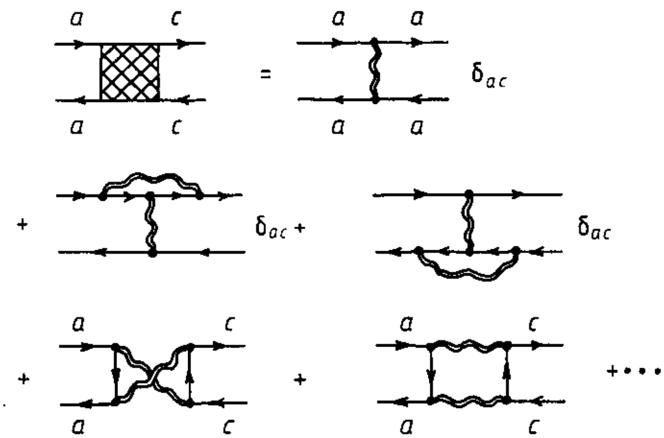

Figure 8.

The obtained equations permit, in principle, the problem of determining the linearized distribution function and conductivity of the CS for arbitrary strong interaction and arbitrary frequencies ω of the external field to be solved. However, to solve equation (50) it is necessary to realize the analytical continuation of this equation from a discrete set of points to some frequency range. Such a problem is extremely complicated even in the case of a weak interaction (Maleev 1970). On the other hand, in the framework of the KE such a problem does not arise when collision integrals are determined. In order to clarify the origin of this statement, let us represent the exact equation (40) for the linearized distribution function in the form of the integral equation

$$-\mathrm{i}\omega f_a^{(1)}(\boldsymbol{p}, k\to 0, \omega) + \frac{z_a e}{\hbar}\frac{\partial f_a^{(0)}(p)}{\partial p_\alpha} E_\alpha(k\to 0, \omega) = \sum_b \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} W_{ab}(\boldsymbol{p}, \boldsymbol{p}', \omega) f_b^{(1)}(\boldsymbol{p}', k\to 0, \omega) \tag{51}$$

$$W_{ab}(\boldsymbol{p}, \boldsymbol{p}', \omega) = \mathrm{i}\omega V \langle\langle \hat{N}_p^a | \hat{N}_{p'}^b \rangle\rangle_\omega \left(\frac{\partial f_a^{(0)}(p')}{\partial \varepsilon_{p'}^b} - \frac{m_b}{\hbar z_b e p'^2} \langle\langle \hat{N}_{p'}^b | p'_\alpha \hat{I}^\alpha \rangle\rangle_\omega \right)^{-1}. \tag{52}$$

The integral equation (51) is the most important result of this paper. It should be considered as the KE in a weakly non-uniform electric field with the exact inclusion of interparticle interaction and frequency dispersion. In this case the right-hand side of equation (51) represents the sum of the linearized collision integrals. Application of the TLR makes it possible to derive the exact equation (52) for the functions $W_{ab}(\boldsymbol{p}, \boldsymbol{p}', \omega)$ determining the collision integrals. Using analogy to the KE method, the meaning of the introduction of W_{ab} is in the possibility of applying direct perturbation theory with respect to interparticle interaction to calculate these functions. However, according to definition (52), the exact calculation of W_{ab} is reduced to the calculation of C_{ab} (equation (48)). Thus, strictly speaking, the problem of summing the series with respect to parameters is conserved.

On the other hand, when determining the collision integrals within the limits of the KE method, expansion in a perturbation theory series is connected only with the parameters γ_{ab} and α_{ab} (Silin 1971, Klimontovich 1975). This means that when expanding the numerator and denominator on the right-hand side of equation (52) in a perturbation theory series with respect to a parameter, a compensation of the series terms with respect to this parameter occurs. Naturally, a proof in general form of such compensation of the series terms with respect to the parameter $\varkappa_{ab}$ is equivalent to a calculation of the functions $C_{ab}(\boldsymbol{p}, \boldsymbol{p}', \omega)$.

Let us consider the consequences of the proposed assumption. Firstly, with respect to the above, the function W_{ab} can be calculated from the formula

$$W_{ab}(\boldsymbol{p}, \boldsymbol{p}', \omega) = \mathrm{i}\omega \left(\frac{\partial f_b^{(0)}(p')}{\partial \varepsilon_{p'}^{b}} \right)^{-1} VC_{ab}^{(1)}(\boldsymbol{p}, \boldsymbol{p}', \omega) \tag{53}$$

where the superscript to the function C_{ab} indicates the order of the perturbation theory with respect to $\varkappa_{ab}$. Substitution of equation (53) in equation (51) should result in the equation being completely equivalent to the result of the KE method. Let us now prove this for the example of a weakly non-ideal plasma, assuming the validity of the conditions

$$\alpha_{ab} \ll 1 \qquad \gamma_{ab} \ll 1. \tag{54}$$

4. Linearized kinetic equation for a weakly non-ideal plasma in an electric field of arbitrary frequency

Let us consider a completely ionized electron–ion plasma with a weak interparticle interaction (conditions (54)) with arbitrary degeneracy of particles. Due to conditions (54) we shall use as the functions $g_a(\boldsymbol{p}, \mathrm{i}\omega_m)$ the single-particle Green functions $g_a^{(0)}(\boldsymbol{p}, \mathrm{i}\omega_m)$ of free particles:

$$g_a^{(0)}(\boldsymbol{p}, \mathrm{i}\omega_m) = (\mathrm{i}\omega_m - \varepsilon_p^a + \mu_a)^{-1}. \tag{55}$$

For completeness the ions will be considered as fermions, so that in equation (55) $\omega_m = (2m+1)k_B T\pi$. Expansion in a perturbation theory series will be carried out with respect to the screened potential of the Coulomb interaction $u_{ab}^{s}(q, \mathrm{i}\Omega)$ (double wavy line in figure 8) (Klyuchnikov and Trigger 1982). The explicit form of the potential $u_{ab}^{s}(q, \mathrm{i}\Omega)$ in the case under consideration can be easily derived using the following exact expression (Kraeft *et al* 1986):

$$u_{ab}^{s}(q, \mathrm{i}\Omega) = u_{ab}(q) + \sum_{cd} u_{ac}(q)\Pi_{cd}(q, \mathrm{i}\Omega)u_{db}^{s}(q, \mathrm{i}\Omega). \tag{56}$$

If conditions (54) are satisfied, the polarization operators $\Pi_{ie}(q, \mathrm{i}\Omega)$ and $\Pi_{ei}(q, \mathrm{i}\Omega)$ cannot be taken into account (Klyuchnikov and Trigger 1976) while the operators $\Pi_{aa}(q, \mathrm{i}\Omega)$ can be considered in the lower approximation with respect to interaction (random-phase approximation, RPA):

$$\Pi_{aa}(q, \mathrm{i}\Omega) \approx \Pi_{aa}^{\mathrm{RPA}}(q, \mathrm{i}\Omega). \tag{57}$$

As a result, we obtain from equation (56)

$$u^{s}_{ab}(q, \mathrm{i}\Omega) = \frac{u_{ab}(q)}{\varepsilon(q, \mathrm{i}\Omega)} \tag{58}$$

$$\varepsilon(q, \mathrm{i}\Omega) = 1 - \sum_{a} u_{aa}(q)\Pi^{\mathrm{RPA}}_{aa}(q, \mathrm{i}\Omega). \tag{59}$$

Let us pass to direct calculation of the function $C^{(1)}_{ab}(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega_n)$. In the case under consideration (conditions (54)) it is sufficient to take into account the diagrams in figure 9 (Perel and Eliashberg 1962). With respect to the laws of the diagram technique (Abrikosov *et al* 1962), we obtain

$$C^{(1)}_{ab}(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega_n) = A^{(1)}_{ab}(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega_n) + A^{(2)}_{ab}(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega_n) + A^{(3)}_{ab}(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega_n) \tag{60}$$

$$\begin{aligned} A^{(1)}_{ab}(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega_n) &= k_{\mathrm{B}}^2 T^2 \sum_{\omega_m} \sum_{\alpha_l} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} u^{s}_{aa}(q, \mathrm{i}\alpha_l) \\ &\times (g^{(0)}_a(\boldsymbol{p}, \mathrm{i}\omega_m + \mathrm{i}\Omega_n) g^{(0)}_a(\boldsymbol{p}, \mathrm{i}\omega_m + \mathrm{i}\Omega_n) g^{(0)}_a(\boldsymbol{p}, \mathrm{i}\omega_m) \\ &\times g^{(0)}_a(\boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\omega_m + \mathrm{i}\Omega_n + \mathrm{i}\alpha_l) + g^{(0)}_a(\boldsymbol{p}, \mathrm{i}\omega_m + \mathrm{i}\Omega_n) \end{aligned}$$

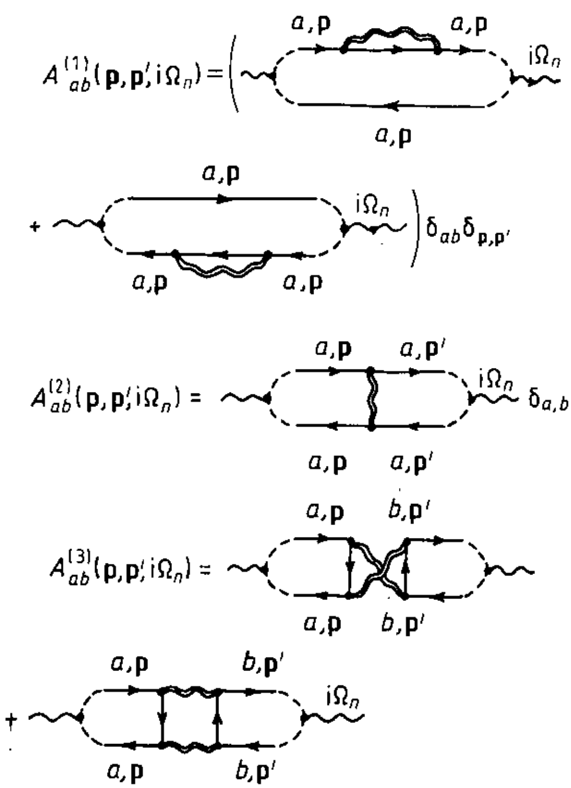

Figure 9.

$$\times g_a^{(0)}(\boldsymbol{p}, \mathrm{i}\omega_m) g_a^{(0)}(\boldsymbol{p}, \mathrm{i}\omega_m) g_a^{(0)}(\boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\omega_m+\mathrm{i}\alpha_l))\delta_{a,b}\delta_{\boldsymbol{p},\boldsymbol{p}'}$$
$$= \frac{k_B T}{(\mathrm{i}\Omega_n)^2} \sum_{\alpha_l} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} u_{aa}^{\mathrm{s}}(q, \mathrm{i}\alpha_l)(Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\alpha_l+\mathrm{i}\Omega_n)$$
$$+ Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\alpha_l-\mathrm{i}\Omega_n) - 2Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\alpha_l))\delta_{a,b}\delta_{\boldsymbol{p},\boldsymbol{p}'} \tag{61}$$

$$A_{ab}^{(2)}(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega_n)$$
$$= \frac{k_B^2 T^2}{V} \sum_{\omega_m} \sum_{\alpha_l} u_{aa}^{\mathrm{s}}(\boldsymbol{p}'-\boldsymbol{p}, \mathrm{i}\alpha_l) g_a^{(0)}(\boldsymbol{p}, \mathrm{i}\omega_m+\mathrm{i}\Omega_n)$$
$$\times g_a^{(0)}(\boldsymbol{p}, \mathrm{i}\omega_m) g_a^{(0)}(\boldsymbol{p}', \mathrm{i}\omega_m+\mathrm{i}\Omega_n+\mathrm{i}\alpha_l) g_a^{(0)}(\boldsymbol{p}', \mathrm{i}\omega_m+\mathrm{i}\alpha_l)\delta_{a,b}$$
$$= \frac{k_B T}{(\mathrm{i}\Omega_n)^2 V} \sum_{\alpha_l} u_{aa}^{\mathrm{s}}(\boldsymbol{p}'-\boldsymbol{p}, \mathrm{i}\alpha_l)(2Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\alpha_l) - Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\alpha_l+\mathrm{i}\Omega_n)$$
$$- Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\alpha_l-\mathrm{i}\Omega_n))\delta_{a,b} \tag{62}$$

$$A_{ab}^{(3)}(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega_n)$$
$$= \frac{k_B^3 T^3}{V} \sum_{\omega_m} \sum_{\omega_k} \sum_{\omega_l} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} u_{ab}^{\mathrm{s}}(q, \mathrm{i}\alpha_l) u_{ab}^{\mathrm{s}}(q, \mathrm{i}\alpha_l+\mathrm{i}\Omega_n)$$
$$\times g_a^{(0)}(\boldsymbol{p}, \mathrm{i}\omega_m+\mathrm{i}\Omega_n) g_a^{(0)}(\boldsymbol{p}, \mathrm{i}\omega_m) g_a^{(0)}(\boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\omega_m+\mathrm{i}\alpha_l+\mathrm{i}\Omega_n)$$
$$\times g_g^{(0)}(\boldsymbol{p}', \mathrm{i}\omega_k+\mathrm{i}\Omega_n) g_g^{(0)}(\boldsymbol{p}', \mathrm{i}\omega_k)$$
$$\times (g_b^{(0)}(\boldsymbol{p}'-\boldsymbol{q}, \mathrm{i}\omega_k-\mathrm{i}\alpha_l) + g_b^{(0)}(\boldsymbol{p}'+\boldsymbol{q}, \mathrm{i}\omega_k+\mathrm{i}\alpha_l+\mathrm{i}\Omega_n))$$
$$= \frac{k_B T}{(\mathrm{i}\Omega_n)^2 V} \sum_{\alpha_l} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} u_{ab}^{\mathrm{s}}(\boldsymbol{q}, \mathrm{i}\alpha_l) u_{ab}^{\mathrm{s}}(q, \mathrm{i}\alpha_l+\mathrm{i}\Omega_n)$$
$$\times (Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\alpha_l+\mathrm{i}\Omega_n) - Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\alpha_l))$$
$$\times (Q_{bb}^T(\boldsymbol{p}', \boldsymbol{p}'-\boldsymbol{q}, -\mathrm{i}\alpha_l) - Q_{bb}^T(\boldsymbol{p}', \boldsymbol{p}'-\boldsymbol{q}, -\mathrm{i}\alpha_l-\mathrm{i}\Omega_n)$$
$$+ Q_{bb}^T(\boldsymbol{p}', \boldsymbol{p}'+\boldsymbol{q}, \mathrm{i}\alpha_l+\mathrm{i}\Omega_n) - Q_{bb}^T(\boldsymbol{p}', \boldsymbol{p}'+\boldsymbol{q}, \mathrm{i}\alpha_l)) \tag{63}$$

$$Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}', \mathrm{i}\Omega) = \frac{f_a^{(0)}(p) - f_a^{(0)}(p')}{\varepsilon_p^a - \varepsilon_{p'}^a + \mathrm{i}\Omega} \tag{64}$$

$$\Pi_{aa}^{\mathrm{RPA}}(q, \mathrm{i}\Omega) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} Q_{aa}^T(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \mathrm{i}\Omega). \tag{65}$$

Here and below $f_a^{(0)}(p)$ is the Fermi-Dirac distribution function for particles of type a.

As seen from equations (61)-(63), it is necessary to define the analytical continuation of the function

$$k_B T \sum_{\alpha_l} \varphi(\mathrm{i}\alpha)\psi(\mathrm{i}\alpha_l+\mathrm{i}\Omega) \tag{66}$$

with respect to the variable $\mathrm{i}\Omega_n$ from discrete points on the imaginary axis ($\Omega_n > 0$) to the real axis. As is shown by Perel and Eliashberg (1962)

$$k_B T \sum_{\alpha_l} \varphi(\mathrm{i}\alpha_l)\psi(\mathrm{i}\alpha_l+\mathrm{i}\Omega_n)$$
$$= \int_{-\infty}^{\infty} \frac{\mathrm{d}\alpha}{2\pi} \coth\left(\frac{\alpha}{2k_B T}\right)(\psi^{\mathrm{R}}(\alpha+\mathrm{i}\Omega_n) \,\mathrm{Im}\, \varphi^{\mathrm{R}}(\alpha) + \varphi^{\mathrm{A}}(\alpha-\mathrm{i}\Omega_n) \,\mathrm{Im}\, \psi^{\mathrm{R}}(\alpha)). \tag{67}$$

The analytical continuation is now reduced to substituting $\mathrm{i}\Omega_n$ by $\hbar(\omega+\mathrm{i}\delta)$, $\delta=+0$; $\varphi^{\mathrm{R(A)}}$ is the retarded (advanced) Green function. For real values of α

$$(\varphi^{\mathrm{R}}(\alpha))^*=\varphi^{\mathrm{A}}(\alpha). \tag{68}$$

Therefore, the linearized KE for a weakly non-ideal plasma takes the form

$$\begin{aligned}
&-\mathrm{i}\omega f_a^{(1)}(p,k\to 0,\omega)+\frac{z_a e}{\hbar}\frac{\partial f_a^{(0)}(p)}{\partial p_\alpha}E_\alpha(k\to 0,\omega)\\
&\quad=\frac{\mathrm{i}}{\hbar\omega}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega_1}{2\pi}\coth\left(\frac{\hbar\omega_1}{2k_{\mathrm{B}}T}\right)\\
&\quad\times[(Q_{aa}^{\mathrm{R}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1+\omega)+Q_{aa}^{\mathrm{A}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1-\omega))\,\mathrm{Im}\,u_{aa}^{\mathrm{SR}}(q,\omega_1)\\
&\quad+(u_{aa}^{\mathrm{SR}}(q,\omega_1+\omega)+u_{aa}^{\mathrm{SA}}(q,\omega_1-\omega))\,\mathrm{Im}\,Q_{aa}^{\mathrm{R}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1)\\
&\quad-2\,\mathrm{Im}(u_{aa}^{\mathrm{SR}}(q,\omega_1)Q_{aa}^{\mathrm{R}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1))](\psi_a(\boldsymbol{p},\omega)-\psi_a(\boldsymbol{p}+\boldsymbol{q},\omega))\\
&\quad+\frac{\mathrm{i}}{\hbar\omega}\sum_b\int\frac{\mathrm{d}^3p'}{(2\pi)^3}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega_1}{2\pi}\coth\left(\frac{\hbar\omega_1}{2k_{\mathrm{B}}T}\right)\\
&\quad\times[(u_{ab}^{\mathrm{SR}}(q,\omega_1+\omega)Q_{aa}^{\mathrm{R}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1+\omega)+u_{ab}^{\mathrm{SA}}(q,\omega_1-\omega)\\
&\quad\times Q_{aa}^{\mathrm{A}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1-\omega))\,\mathrm{Im}(u_{ab}^{\mathrm{SR}}(q,\omega_1)Q_{bb}^{\mathrm{R}}(\boldsymbol{p}'-\boldsymbol{q},\boldsymbol{p}',\omega_1))\\
&\quad+(u_{ab}^{\mathrm{SR}}(q,\omega_1+\omega)Q_{bb}^{\mathrm{R}}(\boldsymbol{p}'-\boldsymbol{q},\boldsymbol{p}',\omega_1+\omega)+u_{ab}^{\mathrm{SA}}(q,\omega_1-\omega)\\
&\quad\times Q_{bb}^{\mathrm{A}}(\boldsymbol{p}'-\boldsymbol{q},\boldsymbol{p}',\omega_1-\omega))\,\mathrm{Im}(u_{ab}^{\mathrm{SR}}(q,\omega_1)Q_{aa}^{\mathrm{R}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1))\\
&\quad-(u_{ab}^{\mathrm{SR}}(q,\omega_1+\omega)Q_{aa}^{\mathrm{R}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1+\omega)Q_{bb}^{\mathrm{R}}(\boldsymbol{p}'-\boldsymbol{q},\boldsymbol{p}',\omega_1+\omega)\\
&\quad+u_{ab}^{\mathrm{SA}}(q,\omega_1-\omega)Q_{aa}^{\mathrm{A}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1-\omega)Q_{bb}^{\mathrm{A}}(\boldsymbol{p}'-\boldsymbol{q},\boldsymbol{p}',\omega_1-\omega))\\
&\quad\times\mathrm{Im}\,u_{ab}^{\mathrm{SR}}(q,\omega_1)-(u_{ab}^{\mathrm{SR}}(q,\omega_1+\omega)+u_{ab}^{\mathrm{SA}}(q,\omega_1-\omega))\\
&\quad\times\mathrm{Im}(u_{ab}^{\mathrm{SR}}(q,\omega_1)Q_{aa}^{\mathrm{R}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1)Q_{bb}^{\mathrm{R}}(\boldsymbol{p}'-\boldsymbol{q},\boldsymbol{p}',\omega_1))]\\
&\quad\times(\psi_b(\boldsymbol{p}',\omega)-\psi_b(\boldsymbol{p}'-\boldsymbol{q},\omega)).
\end{aligned}\tag{69}$$

In equation (69)

$$Q_{aa}^{\mathrm{R(A)}}(\boldsymbol{p},\boldsymbol{p}',\omega)=\frac{f_a^{(0)}(p)-f_a^{(0)}(p')}{\varepsilon_p^a-\varepsilon_{p'}^a+\hbar(\omega\pm\mathrm{i}0)} \tag{70}$$

$$f_a^{(1)}(\boldsymbol{p},k\to 0,\omega)=\frac{\partial f_a^{(0)}(p)}{\partial\varepsilon_p^a}\psi_a(\boldsymbol{p},\omega). \tag{71}$$

For comparison of the results of the KE and TLR for a weakly non-ideal plasma, let us consider the limiting cases: $\omega\to 0$ and $\omega\gg\bar{\nu}_{ab}$.

At the static limit $\omega\to 0$ the right-hand side of equation (69) takes the form

$$\begin{aligned}
&\lim_{\omega\to 0}\sum_b\int\frac{\mathrm{d}^3p'}{(2\pi)^3}W_{ab}(\boldsymbol{p},\boldsymbol{p}',\omega)f_b^{(1)}(\boldsymbol{p}',\boldsymbol{k}\to 0,\omega)\\
&\quad=-\frac{1}{k_{\mathrm{B}}T}\sum_b\int\frac{\mathrm{d}^3q}{(2\pi)^3}\int\frac{\mathrm{d}^3p'}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega'}{2\pi}\sinh^{-2}\left(\frac{\hbar\omega_1}{2k_{\mathrm{B}}T}\right)\\
&\quad\times|u_{ab}^{\mathrm{SR}}(q,\omega_1)|^2\,\mathrm{Im}\,Q_{aa}^{\mathrm{R}}(\boldsymbol{p},\boldsymbol{p}+\boldsymbol{q},\omega_1)\,\mathrm{Im}\,Q_{bb}^{\mathrm{R}}(\boldsymbol{p}'-\boldsymbol{q},\boldsymbol{p}',\omega_1)\\
&\quad\times[\psi_a(\boldsymbol{p},\omega\to 0)-\psi_a(\boldsymbol{p}+\boldsymbol{q},\omega\to 0)\\
&\quad+\psi_b(\boldsymbol{p}',\omega\to 0)-\psi_b(\boldsymbol{p}'-\boldsymbol{q},\omega\to 0)]
\end{aligned}\tag{72}$$

with equations (70) and (71) taken into account this is completely equivalent to the linearized Lenard–Balesku collision integrals (Silin 1971, Klimontovich 1975).

In the high-frequency case $\omega \gg \bar{\nu}_{ab}$ the right-hand side of equation (69) can be considered as small and the functions included in it can be substituted by the following expression:

$$\psi_a(\boldsymbol{p}, \omega) = -\frac{\mathrm{i}\hbar z_a e}{m_a \omega} p_\alpha E_\alpha(k \to 0, \omega). \tag{73}$$

Then

$$\begin{aligned}
&f_a^{(1)}(\boldsymbol{p}, k \to 0, \omega)\\
&\quad= -\frac{\mathrm{i} z_a e}{\hbar\omega}\frac{\partial f_a^{(0)}(p)}{\partial p_\alpha} E_\alpha(k \to 0, \omega) - \frac{z_a e}{m_a \omega^2}\\
&\quad\times \int \frac{\mathrm{d}^3 q}{(2\pi)^3} q_\alpha E_\alpha(k \to 0, \omega) \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_1}{2\pi} \coth\left(\frac{\hbar\omega_1}{2k_\mathrm{B}T}\right)\\
&\quad\times [(Q_{aa}^\mathrm{R}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1+\omega) + Q_{aa}^\mathrm{A}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1-\omega)) \operatorname{Im} u_{aa}^\mathrm{SR}(q, \omega_1)\\
&\quad+ (u_{aa}^\mathrm{SR}(q, \omega_1+\omega) + u_{aa}^\mathrm{SA}(q, \omega_1-\omega)) \operatorname{Im} Q_{aa}^\mathrm{R}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1)\\
&\quad- 2\operatorname{Im}(u_{aa}^\mathrm{SR}(q, \omega_1) Q_{aa}^\mathrm{k}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1))]\\
&\quad+ \sum_b \frac{z_b e}{m_b \omega^2} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} q_\alpha E_\alpha(k \to 0, \omega) \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_1}{2\pi} \coth\left(\frac{\hbar\omega_1}{2k_\mathrm{B}T}\right)\\
&\quad\times [(u_{ab}^\mathrm{SR}(q, \omega_1+\omega) Q_{aa}^\mathrm{R}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1+\omega) + u_{ab}^\mathrm{SA}(q, \omega_1-\omega)\\
&\quad\times Q_{aa}^\mathrm{A}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1-\omega)) \operatorname{Im}(u_{ab}^\mathrm{SR}(q, \omega_1)\Pi_{bb}^\mathrm{R}(q, \omega_1))\\
&\quad+ (u_{ab}^\mathrm{SR}(q, \omega_1+\omega)\Pi_{bb}^\mathrm{R}(q, \omega_1+\omega) + u_{ab}^\mathrm{SA}(q, \omega_1-\omega)\Pi_{bb}^\mathrm{A}(q, \omega_1-\omega))\\
&\quad\times \operatorname{Im}(u_{ab}^\mathrm{SR}(q, \omega_1) Q_{aa}^\mathrm{R}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1)) - (u_{ab}^\mathrm{SR}(q, \omega_1+\omega)\\
&\quad\times Q_{aa}^\mathrm{R}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1+\omega)\Pi_{bb}^\mathrm{R}(q, \omega_1+\omega) + u_{ab}^\mathrm{SA}(q, \omega_1-\omega)\\
&\quad\times Q_{au}^\mathrm{A}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1-\omega)\Pi_{bb}^\mathrm{A}(q, \omega_1-\omega)) \operatorname{Im} u_{ab}^\mathrm{SR}(q, \omega_1)\\
&\quad- (u_{ab}^\mathrm{SR}(q, \omega_1+\omega) + u_{ab}^\mathrm{SA}(q, \omega_1+\omega)) \operatorname{Im}(u_{ab}^\mathrm{SR}(q, \omega_1)\\
&\quad\times Q_{aa}^\mathrm{R}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1)\Pi_{bb}^\mathrm{R}(q, \omega_1))].
\end{aligned} \tag{74}$$

For a classical plasma ($\hbar \to 0$), neglecting screening effects, equation (74) corresponds to the results obtained by Silin (1961) in the framework of the KE method. Using equation (74) for the linearized distribution function, one can easily derive the explicit form of the high-frequency conductivity of a weakly non-ideal plasma ($|z_e|/m_e \gg |z_i|m_i$):

$$\sigma(\omega) = \frac{\mathrm{i}\omega_p^2}{4\pi\omega}\left(1 + \gamma(\omega) - \frac{\mathrm{i}\nu(\omega)}{\omega}\right) \tag{75}$$

$$\begin{aligned}
\nu(\omega) = &\frac{1 - \exp(-\hbar\omega/k_\mathrm{B}T)}{\sigma m_e n_e \hbar\omega} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} q^2 |u_{ei}(q)|^2\\
&\times \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_1}{2\pi} (S_{ii}(q, \omega_1) S_{ee}(q, \omega-\omega_1) - S_{ei}(q, \omega_1) S_{ie}(q, \omega-\omega_1))
\end{aligned} \tag{76}$$

$$\gamma(\omega)=\frac{1}{3m_en_e\omega^2}\int\frac{d^3q}{(2\pi)^3}q^2|u_{ei}(q)|^2\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}$$
$$\times[S_{ii}(q,\omega_1)\,\mathrm{Re}(L^R_{ee}(q,\omega-\omega_1)-L^R_{ee}(q,\omega_1))$$
$$+S_{ee}(q,\omega_1)\,\mathrm{Re}(L^R_{ii}(q,\omega+\omega_1)-L^R_{ii}(q,\omega_1))$$
$$-S_{ie}(q,\omega_1)\,\mathrm{Re}(L^R_{ei}(q,\omega+\omega_1)-L^R_{ei}(q,\omega_1))$$
$$-S_{ei}(q,\omega_1)\,\mathrm{Re}(L^R_{ie}(q,\omega-\omega_1)-L^R_{ie}(q,\omega_1))] \qquad (77)$$

$$S_{ab}(q,\omega)=-2\hbar\left[1-\exp\left(-\frac{\hbar\omega}{k_BT}\right)\right]^{-1}\mathrm{Im}\,L^R_{ab}(q,\omega) \qquad (78)$$

$$L^R_{aa}(q,\omega)=\frac{\Pi^R_{aa}(q,\omega)(1-u_{bb}(q)\Pi^R_{bb}(q,\omega))}{\varepsilon^R(q,\omega)} \qquad (79)$$

$$L^R_{ab}(q,\omega)=L^R_{ba}(q,\omega)=\frac{u_{ab}(q)\Pi^R_{aa}(q,\omega)\Pi^R_{bb}(q,\omega)}{\varepsilon^R(q,\omega)}. \qquad (80)$$

In equations (79) and (80) the function $\varepsilon^R(q,\omega)$ is determined by formula (59); $a\neq b$.

Equations (75)-(80) for high-frequency conductivity are completely equivalent to the results of Perel' and Eliashberg (1962) for gaseous plasma and Bobrov and Trigger (1984) for metallic plasma in the framework of the TLR and, with respective approximations, to the results (Silin 1961, Klimontovich 1975) obtained by the KE method.

Thus, assumption (53) on the form of the function $W_{ab}(\boldsymbol{p},\boldsymbol{p}',\omega)$ leads to the known results of the KE method in the frequency ranges $\omega\to 0$ and $\omega\gg\bar{\nu}_{ab}$. Therefore, equation (69) can be considered as the quantum linearized equation for a weakly non-ideal completely ionized plasma in an electric field of arbitrary frequency ω.

However, the question of the validity of assumption (53) still remains open even in the case of a weakly non-ideal plasma. Since a consistent calculation of the function C_{ab} in the framework of the diagram technique has not yet been carried out, let us try to examine the consequences of assumption (53).

5. On the condition of applicability of the KE for a weakly non-ideal plasma

Let us introduce into consideration the function

$$\nu_a(p,\omega)=-\frac{m_a}{z_aep^2}\int\frac{d^3p'}{(2\pi)^3}\sum_b\frac{\hbar z_bep_\alpha p'_\alpha}{m_b}W_{ba}(\boldsymbol{p}',\boldsymbol{p},\omega). \qquad (81)$$

Using diagram expansions, one can easily verify (see, for example, equations (60)-(65)) that

$$C_{ab}(\boldsymbol{p},\boldsymbol{p}',\mathrm{i}\Omega_n)=C_{ba}(\boldsymbol{p}',\boldsymbol{p},\mathrm{i}\Omega_n). \qquad (82)$$

Therefore,

$$\nu_a(p,\omega)=\mathrm{i}\omega\langle\langle\hat{N}^a_p|p_\alpha\hat{I}^\alpha\rangle\rangle_\omega\left(-\frac{z_aep}{\hbar}\frac{\partial f^{(0)}_a(p)}{\partial p}+\langle\langle\hat{N}^a_p|p_\beta\hat{I}^\beta\rangle\rangle_\omega\right)^{-1}. \qquad (83)$$

With equation (83) taken into account, one can write the exact expression (10) for the linearized distribution function in the form (Bobrov *et al* 1989, Bobrov and Trigger 1990)

$$-\mathrm{i}\omega f^{(1)}_a(\boldsymbol{p},k\to 0,\omega)+\frac{z_ae}{\hbar}\frac{\partial f^{(0)}_a(p)}{\partial p_\alpha}E_\alpha(k\to 0,\omega)=-\nu_a(p,\omega)f^{(1)}_a(\boldsymbol{p},k\to 0,\omega). \qquad (84)$$

According to equation (84), the function $\nu_a(p, \omega)$ represents the effective collision frequency for particles of type a, and is dependent both on the particle momentum and the external field frequency. As is clear from general considerations, at finite values of ω the function $\nu_a(p, \omega)$ is complex, so that the time between collisions is precisely characterized by the function Re $\nu_a(p, \omega)$.

Since equations (54) and (84) are exact and, therefore, equivalent, the equivalence of these equations should not be violated in the correct calculation of the function $W_{ab}(\boldsymbol{p}, \boldsymbol{p}', \omega)$. This criterion can be considered as the necessary condition for correct calculation of the function $W_{ab}(\boldsymbol{p}, \boldsymbol{p}', \omega)$.

Thus, the linearized KE (equation (69)) for a weakly non-ideal plasma holds true if its solution has the form

$$f_a^{(1)}(\boldsymbol{p}, k \to 0, \omega) = -\frac{z_a e}{\hbar} \frac{\partial f_a^{(0)}(p)}{\partial p_\alpha} E_\alpha(k \to 0, \omega)(-\mathrm{i}\omega + \nu_a(p, \omega))^{-1} \tag{85}$$

where $\nu_a(p, \omega)$, according to equations (40), (60)-(68) and (81), has the form (Bobrov *et al* 1989)

$$\begin{aligned}
\nu_a(p, \omega) = &-\frac{\mathrm{i}\hbar}{m_a p \omega} \left(\frac{\partial f_a^{(0)}(p)}{\partial p}\right)^{-1} \sum_b \left(1 - \frac{m_a z_b}{m_b z_a}\right) \\
&\times \int \frac{\mathrm{d}^3 q}{(2\pi)^3} p_\alpha q_\alpha \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_1}{2\pi} \coth\left(\frac{\hbar\omega_1}{2k_\mathrm{B} T}\right) \\
&\times [(u_{ab}^{\mathrm{SR}}(q, \omega_1 + \omega) Q_{aa}^{\mathrm{R}}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1 + \omega) \Pi_{bb}^{\mathrm{R}}(q, \omega_1 + \omega) \\
&+ u_{ab}^{\mathrm{SA}}(q, \omega_1 - \omega) Q_{aa}^{\mathrm{A}}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1 - \omega) \Pi_{bb}^{\mathrm{A}}(q, \omega_1 - \omega)) \,\mathrm{Im}\, u_{ab}^{\mathrm{SR}}(q, \omega_1) \\
&+ (u_{ab}^{\mathrm{SR}}(q, \omega_1 + \omega) + u_{ab}^{\mathrm{SA}}(q, \omega_1 - \omega)) \,\mathrm{Im}(u_{ab}^{\mathrm{SR}}(q, \omega_1) \\
&\times Q_{aa}^{\mathrm{R}}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1) \Pi_{bb}^{\mathrm{R}}(q, \omega_1)) - (u_{ab}^{\mathrm{SR}}(q, \omega_1 + \omega) \Pi_{bb}^{\mathrm{R}}(q, \omega_1 + \omega) \\
&+ u_{ab}^{\mathrm{SA}}(q, \omega_1 - \omega) \Pi_{bb}^{\mathrm{A}}(q, \omega_1 - \omega)) \,\mathrm{Im}(u_{ab}^{\mathrm{SR}}(q, \omega_1) Q_{aa}^{\mathrm{R}}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1)) \\
&- (u_{ab}^{\mathrm{SR}}(q, \omega_1 + \omega) Q_{aa}^{\mathrm{R}}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1 + \omega) + u_{ab}^{\mathrm{SA}}(q, \omega_1 - \omega) \\
&\times Q_{aa}^{\mathrm{A}}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1 - \omega)) \,\mathrm{Im}(u_{ab}^{\mathrm{SR}}(q, \omega_1) \Pi_{bb}^{\mathrm{R}}(q, \omega_1))].
\end{aligned} \tag{86}$$

Let us immediately emphasize that in the high-frequency case $\omega \gg \bar{\nu}_{ab}$ we arrive at equation (74) when calculating $f_a^{(1)}(\boldsymbol{p}, k \to 0, \omega)$ with the accuracy of first-order terms with respect to $\nu_a(p, \omega)/\omega$.

In the static case $\omega \to 0$

$$\begin{aligned}
\nu_a(p) &= \lim_{\omega \to 0} \nu_a(p, \omega) \\
&= \frac{2\hbar^2}{m_a p k_\mathrm{B} T} \left(\frac{\partial f_a^{(0)}(p)}{\partial p}\right)^{-1} \sum_b \left(1 - \frac{m_a z_b}{m_b z_a}\right) \\
&\quad \times \int \frac{\mathrm{d}^3 q}{(2\pi)^3} p_\alpha q_\alpha \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_1}{2\pi} \sinh^{-2}\left(\frac{\hbar\omega_1}{2k_\mathrm{B} T}\right) |u_{ab}^{\mathrm{SR}}(q, \omega_1)|^2 \\
&\quad \times \mathrm{Im}\, Q_{aa}^{\mathrm{R}}(\boldsymbol{p}, \boldsymbol{p}+\boldsymbol{q}, \omega_1) \,\mathrm{Im}\, \Pi_{bb}^{\mathrm{R}}(q, \omega_1).
\end{aligned} \tag{87}$$

Thus, in the static case coincidence of the KE results (equation (69)) with equation (85) takes place only if one considers a system of electrons in the medium of immovable scatterers (ions) without taking account of the electron–electron collision integral in the KE. Therefore, the results obtained on the basis of the KE method can be considered as reliable when considering such systems as liquid metal, weakly ionized plasma, etc.

In the general case, as is clear from the results obtained, the construction of a consistent non-contradictory perturbation theory for the examination of the distribution function in a weak electric field requires the diagram equations (48)–(50) to be solved.

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